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The Gravitational Instability of the Vacuum: Insight into the Cosmological Constant Problem

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ABSTRACT

A mechanism for suppressing the cosmological constant is developed, based on an analogy with a superconducting phaseshift in which free fermions coupled perturbatively to a weak gravitational field are in an unstable false vacuum state. The coupling of the fermions to the gravitational field generates fermion condensates with zero momentum and a phase transition induces a nonperturbative transition to a true vacuum state by producing a positive energy gap Δ in the vacuum energy, identified with $\sqrt{\Lambda}$, where Λ is the cosmological constant. In the strong coupling limit a large cosmological constant induces a period of inflation in the early universe, followed by a weak coupling limit in which $\sqrt{\Lambda}$ vanishes exponentially fast as the universe expands due to the dependence of the energy gap on the density of Fermi surface fermions, $D(\epsilon)$, predicting a small cosmological constant in the present universe.

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Contents

1 Introduction

The cosmological constant problem centers around two questions:

1) Why is the cosmological constant observed to be much smaller than the expected Planckian value?

2) If we choose the bare cosmological constant to be the observed value today, what is the mechanism which stabilizes it under quantum corrections [1, 2]?

Many investigators have proposed solutions ranging from a rolling scalar field to the anthropic principle[3, 4, 5, 6]. Moreover, it can be argued that the greatest irony of the cosmological constant problem is the inflationary paradigm[7, 8, 9]. The onset of the inflationary epoch is completely dominated with a fluid that is either a pure cosmological constant or a scalar field whose potential is extremely flat, mimicking a cosmological constant. Clearly this period of inflation has to end for successful structure formation. However, whatever mechanism is responsible for this exit from inflation should be a clue as to how the cosmological constant is relaxed at all times including today.

In the following, we shall propose a mechanism to solve the cosmological constant problem, based on a simple idea analogous to the microscopic realization of superconductivity. We argue that the perturbative vacuum state in the presence or absence of a cosmological constant is gravitationally unstable and it is energetically favorable for the vacuum associated with the effective cosmological constant to release all of its energy into the production of condensates, bound states of the free fermions. The formation of condensates leads to a non-perturbative true ground state. Similarly, Bose-Einstein condensates have been proposed by [10, 11, 12, 13, 14] as a viable alternative for a fundamental inflaton scalar field, because they can have a nonzero potential sufficiently flat to lead to inflation. Condensates also enjoy the property of ending inflation since they can vanish in the infra-red (at late times).

In the absence of a gravitational interaction between fermions the Minkowski flat spacetime has a zero cosmological constant. When the gravitational interaction is switched on, the Minkowski spacetime vacuum becomes unstable, and the universe enters into a superfluid phase of fermion condensates. Below a critical phase transition temperature, T_c, the binding energy of a pair of fermions causes the opening of a positive energy gap Δ in the ground state (vacuum energy) of the fermion condensate system. The positive energy gap Δ is identified with the square root of the cosmological constant, $\sqrt{\Lambda}$. We shall describe the formation of the non-perturbative energy gap due to the exchange of gravitons between fermions, in analogy with the exchange of phonons between electrons in a crystal structure in a non-relativistic Bardeen, Cooper and Schrieffer model [15]. In an initial strong coupling limit the energy gap Δ or $\sqrt{\Lambda}$ yields a large enough cosmological constant to induce a period of de Sitter inflation. This is followed by a weak coupling limit as the universe accelerates, leading to an exponential suppression of Λ and a "graceful" exit to inflation.

The non-perturbative nature of the vacuum instability and the formation of a vacuum energy gap in the early universe, explains why any attempt to derive the cosmological constant from a *perturbative* quantum field theory calculation leads to an egregious disagreement with the observed value of the vacuum energy, when the latter is identified with dark energy.

This paper is organized as follows: Section II begins with the construction of a Fermi liquid in de Sitter Space. In section III, we consider a non-relativistic derivation of the energy gap displaying both the weak and strong coupling limit of the fermionic condensate and its relation to the exponential suppression of the cosmological constant. In section IV, we discuss a relativistic field theory model of the formation of Λ in the vacuum energy due to gravity and a screened fermion attractive force, based on the Nambu-Jona-Lasinio [17] 4-fermion interaction model. We conclude the paper, in section V, with a summary of the results and a discussion of future directions of research.

2 The de Sitter Fermi Liquid

In this section, we want to draw some similarities between fermions in de Sitter space and fermions in a superconductor. This analogy was made precise in the Nambu-Jona-Lasinio [17, 18] model of the four fermion interaction. Our main point is to derive the density of states from fermions which naturally arise in de Sitter space and derive its dependence on the scale factor. We now proceed to construct a Fermi-surface in de-Sitter space from N-fermions.

Let us recall that de Sitter space is the maximally symmetric solution of the Einstein equations with a positive cosmological constant. It is generated by an isometry group of $SO(4,1)$ and can be seen as a timelike hyperboloid embedded in a $4+1$ dimensional Minkowski space obeying the constraint:

$$R^2 = -T^2 + X_1^2 + X_2^2 + X_3^2 + X_4^2 \tag{1}$$

One can consider a special class of geodesic observers in de Sitter space which corresponds to a fermionic representation. For example, by specifying the observer at the north pole in the positive X_4 direction, one is left with Lorentz generators which leave the observer's worldline invariant under rotations about the axis connecting the poles as well as the boosts in the X_4 direction. These generators form the observer's little group which is $S0(3) \times R$, whose representations correspond to massless fermions. In general the generators of de Sitter can be written as

$$M_{IJ} = -\frac{i}{4}[\Gamma_I, \Gamma_J] \tag{2}$$

, where the gamma matrices obey the Clifford algebra $\{\Gamma_I, \Gamma_J\} = 2\eta_{IJ}$ and $I, J = 0..4$. Let us express the de Sitter generators by indices μ , ν which run from 1 to 3.

$$J_{\mu\nu} = M_{\mu\nu} \quad P_\mu = M_{4\mu} \quad K_\mu = M_{0\mu} \quad H = M_{04} \tag{3}$$

,

$$J_\mu = \begin{pmatrix} \sigma_\mu & 0 \\ 0 & \sigma_\mu \end{pmatrix} \quad P_\mu = \frac{i}{2}\begin{pmatrix} 0 & \sigma_\mu \\ -\sigma_\mu & 0 \end{pmatrix} \tag{4}$$

$$K_\mu = \frac{i}{2}\begin{pmatrix} 0 & \sigma_\mu \\ \sigma_\mu & 0 \end{pmatrix} \quad H = \frac{i}{2}\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} \tag{5}$$

These generators will act on two component spinors and will form a many particle Hilbert space. We can construct the Lagrangian for free massless fermions in de Sitter space.

$$\mathcal{L} = \sqrt{\mathrm{g}}\left(\mathrm{R(e)} + \bar{\psi}\mathrm{e}^{\mu}_{\mathrm{b}}\gamma^{\mathrm{b}}(\mathrm{i}\partial_{\mu} - \frac{1}{2}\omega_{\mu\mathrm{cd}}\mathrm{J}^{\mathrm{cd}})\psi\right) \tag{6}$$

where $\omega_{\mu cd} = e^{\nu}_{d}\left(e_{\nu c,\mu} - \Gamma^{\rho}_{\mu\nu}e_{\rho c}\right)$ are the spin connection coefficients.

The FRW background is given by the metric

$$ds^2 = a^2(\eta)(-d\eta^2 + dx^i dx_i) \tag{7}$$

where η is the conformal time, noting that for de Sitter space $a = \frac{1}{H\eta}$. Choosing for the vierbein, $e_{\mu b} = a\eta_{\mu b}$, we can solve for the spin connection:

$$\omega_{\mu cd} = (\eta_{\mu c}\partial_d - \eta_{\mu d}\partial_c)\ln(a) \tag{8}$$

Upon substitution into the above Lagrangian we obtain

$$\mathcal{L} = \frac{1}{2}(\mathrm{a}^{3/2}\bar{\psi})\mathrm{i}\gamma^{\mu}\partial_{\mu}(\mathrm{a}^{3/2}\psi) \tag{9}$$

and varying with respect to $\bar{\psi}$, we get the equation of motion

$$i\gamma^{\mu}\partial_{\mu}(a^{3/2}\psi) = 0 \tag{10}$$

or, more explicitly

$$\gamma^0[H\psi + \dot{\psi}] + \vec{\sigma}\cdot\vec{\nabla}\psi = 0 \tag{11}$$

where $H = \dot{a}/a$ is the Hubble parameter. Therefore the solution of a massless Dirac particle propagating in de-Sitter space describes plane waves with a dispersion relation

$$\omega_p^2 = k^2 \tag{12}$$

The density of states will therefore grow in an expanding universe, if they are quantized in a co-moving box. * We are interested in the density of states of these fermions in order to make contact with condensation in the following sections. The density of states can directly be obtained from the definition:

$$D(k) = \frac{dN}{dk}\frac{dk}{dE} \tag{13}$$

After some straightforward algebra we get

$$D(k) \sim a^3(t). \tag{14}$$

This result will be of relevance in the following sections.

*We thank Lenny Susskind and B.J Bjorken for clarifying this issue with one of us (SA).

3 Vacuum Instability, Fermion Condensates, the Gap Equation and the Cosmological Constant

The cosmological constant problem can be stated as follows. We have the Einstein gravitational equations

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R - \Lambda_0 g_{\mu\nu} = 8\pi G T_{\mu\nu}, \tag{15}$$

where Λ_0 is the "bare" cosmological constant and

$$T_{\mu\nu} = T^M_{\mu\nu} + T^{\rm vac}_{\mu\nu} \tag{16}$$

Here,

$$T^{\rm vac}_{\mu\nu} = \rho_{\rm vac} g_{\mu\nu} \tag{17}$$

This leads to the definition of an effective cosmological constant

$$\Lambda_{\rm eff} = \Lambda_0 + 8\pi G\rho_{\rm vac} \tag{18}$$

where $\rho_{\rm vac}$ is the vacuum energy density. A calculation of the vacuum density for a cutoff of order the Planck energy leads to a result that is 120 orders of magnitude larger than the observed value [1, 2].

Let us now consider a *non-perturbative* model to solve the cosmological constant problem, based on an analogy with the microphysical realization of superconductivity. We argue that in the absence of gravitational interactions between fermions, Minkowski spacetime is unstable and the cosmological constant $\Lambda = 0$. A *non-perturbative* phase transition to a true vacuum state occurs when the gravitational interaction is taken into account. The fermions form Cooper pair condensates with zero momentum due to the weak gravitational interaction and a screened long-range attractive interaction among the pairs of fermions. The transition to the true vacuum state produces a non-zero cosmological constant and a de Sitter phase of inflation.

We shall describe the phase transition to fermion condensates using a non-relativistic toy model. The Hamiltonian takes the BCS form with $\mathbf{k} = -\mathbf{k}$ [20]:

$$\mathcal{H} = \sum_{\rm k's',ks} \mathcal{E}_{\rm k} c^\dagger_{\rm ks} c_{\rm ks} - \frac{1}{2}\sum_{\rm k,k',s,s'} V_{\rm kk'} c^\dagger_{\rm k's'} c^\dagger_{\rm -ks} c_{\rm -k's'} c_{\rm ks} \tag{19}$$

We perform the transformation to new operators

$$b_k = u_k c_k - v_k c^\dagger_{-k}, \quad b_{-k} = u_k c_{-k} + v_k c^\dagger_k \tag{20}$$

where the b's satisfy anti-commutation relations and $u_k^2 + v_k^2 = 1$. The fermion number operators are $n_k = b^\dagger_k b_k$ and $n_{-k} = b^\dagger_{-k} b_{-k}$.

We must now determine the ground state (vacuum) and set the occupation numbers n_k and n_{-k} equal to zero. We need to determine the energy gap Δ produced by the gap in the vacuum energy in the phase transition to the fermion condensates. The condensates are bound states due to the weak gravitational interaction generated by the exchange of gravitons between fermions and the screened attractive force. This will give

$$\Delta = \sqrt{\Lambda}$$

We can minimize the Hamiltonian energy by diagonalizing $\mathcal{H}$, giving the condition

$$\mathcal{E}_k\left(\frac{1}{4} - x_k^2\right)^{1/2} + x_k \sum_{k'} V_{kk'}\left(\frac{1}{4} - x_{k'}^2\right)^{1/2} = 0 \tag{21}$$

where $u_k = (\frac{1}{2} - x_k)^{1/2}$ and $v_k = (\frac{1}{2} + x_k)^{1/2}$, and $V_{kk'}$ is the interaction matrix associated with the exchange of gravitons and the screened attractive fermion force.

We define the quantity

$$\Delta_k = \sum_{k'} V_{kk'}\left(\frac{1}{4} - x_{k'}^2\right)^{1/2} \tag{22}$$

Then (21) yields

$$x_k = \pm\frac{\mathcal{E}}{2(\mathcal{E}^2 + \Delta_k^2)^{1/2}} \tag{23}$$

By substituting this into (22), we obtain the integral equation for Δ_k:

$$\Delta_k = \frac{1}{2}\sum_{k'} V_{kk'}\frac{\Delta_{k'}}{(\mathcal{E}_{k'}^2 + \Delta_{k'}^2)^{1/2}} \tag{24}$$

We assume the simple model for the interaction matrix:

$$V_{kk'} = V \quad \text{if } |\mathcal{E}_k| < \omega_D,$$

$$V_{kk'} = 0 \quad \text{otherwise} \tag{25}$$

where ω_D ($\hbar = 1$) is the Debye energy and V is a constant. Choosing the minus sign in (23), we obtain for the energy gap

$$\Delta_k = \frac{1}{2}VD\int_{-\omega_D}^{\omega_D} d\mathcal{E}\frac{\Delta}{(\mathcal{E}^2 + \Delta^2)^{1/2}} \tag{26}$$

The solution to this equation is

$$\Delta = \sqrt{\Lambda} = \frac{\omega_D}{\sinh[1/VD]} \tag{27}$$

where D is the fermion density of states defined by the Fermi sphere for N fermions by

$$n_f = \int_{k_0}^{k_f} d^3k D(k) \tag{28}$$

The physical density of fermion states behaves for an expanding universe as

$$D(\omega_k) \sim a^3(t) \tag{29}$$

so that as the universe inflates and $a(t) \to \infty$, we have $D(\omega) \to \infty$. However, VD is independent of the cosmic scale $a(t)$.

In the early universe there is an initial phase in which spacetime is flat (Minkowski) and the fermions do not interact gravitationally. This phase is unstable to gravitational interactions between fermions. There is a phase transition to an inflating de Sitter vacuum, in which we have a *strong coupling* limit $VD \sim 1$ and

$$\Delta_i = \sqrt{\Lambda} \sim \omega_D. \tag{30}$$

In this phase

$$\sqrt{\Lambda} \sim \omega_D \sim M_{\rm PL} \tag{31}$$

where $M_{\rm PL}$ is the Planck mass. As the universe expands exponentially, a weak coupling limit develops when

$$VD \ll 1 \tag{32}$$

which from (27) leads to an exponential suppression of the cosmological constant

$$\Delta_f = \sqrt{\Lambda} = 2\omega_D \exp\biggl(-\frac{1}{VD}\biggr) \tag{33}$$

The weak coupling limit (33) can be interpreted as a weakening of the correlation between the fermions associated with a decay of the vacuum energy into pairs of particles at the end of inflation. As Λ tends to zero the universe enters the radiation dominated phase of an FRW model. We see that the condensate phase can generate enough inflation initially and then produce an exponential suppression of the cosmological constant, leading to a vanishingly small value of Λ in the present universe.

The condensation energy for the weak coupling limit $VD \ll 1$ is given by

$$\mathcal{E}_{\rm cond} \sim -2\omega_D^2 D \exp\biggl(-\frac{2}{VD}\biggr) \sim -\frac{1}{2}D\Lambda \tag{34}$$

As the universe expands from its initial inflationary period, the number of fermions that is affected by the attractive gravitational and screened interaction is a small fraction of the total number of fermions in the universe. We note that $\exp[-2/(VD)]$ has an essential singularity at $V = 0$, which means that while the function and its derivatives vanish as $V \to +0$, they all become infinite as $V \to -0$. This means that we cannot calculate $\mathcal{E}_{\rm cond}$ by using perturbation theory.

In order for our mechanism to produce a suppression of the cosmological constant, we must have a large enough density of fermions D in the de Sitter phase of inflation. Inflation produces enormous numbers of massless, minimally coupled scalar condensates $\phi = \langle\bar{\psi}\psi\rangle$.

The conformal invariance of free Dirac theory implies that there can be no comparable, direct production of fermions. However, it is possible to produce fermions during inflation by allowing them to interact with a massless, minimally coupled scalar or fermion condensate $\langle\bar{\psi}\psi\rangle$ [16]. The physical interpretation is that inflation alters the constraint of energy conservation to permit the spontaneous appearance of a condensate and a fermion-anti-fermion pair, and the fermions do not recombine to make virtual pairs. This mechanism could produce a large number of fermions in the de Sitter space and allow for a non-zero fermion density D when the fermion occupation numbers n_k and $n_{(-k)}$ are zero.

4 Vacuum Instability in the de Sitter Phase

In the previous section, we provided general arguments for the vacuum instability which drives the universe into a de-Sitter inflationary phase. We also demonstrated that during inflation the weakening of correlations between the fermions will lead to an exponential suppression of the cosmological constant. We now present a relativistic model which shows that the perturbative (false) vacuum in de-Sitter space is unstable in the presence of gravitational interactions to a non-perturbative true vacuum state. Instead, the perturbative graviton interaction between fermion pairs drives the fermions into non-perturbative condensate states †. In the case of inflation, these states are the Goldstone bosons corresponding to the broken de Sitter space-time symmetry, which commences with a large and then exponentially suppressed cosmological constant.

Let us illustrate this mechanism with a model which effectively models the formation of Cooper-pairs in a relativistic gravitational context. The important point is that our Lagrangian (9) will get modified by an interaction Lagrangian which takes into account graviton exchange between pairs of fermions. Consider the following theory with a massless, free fermion coupled to gravity. For illustration, we consider the modified version of eq (9):

$$\mathcal{L} = \sqrt{\mathrm{g}}[\mathrm{R} + \sum_{\mathrm{k}=1}^{\mathrm{N}} \bar{\psi}\mathrm{D_a}\gamma^{\mathrm{a}}\psi \quad + \sum_{\mathrm{k}=1}^{\mathrm{N}} \frac{\mathrm{G}}{2\mathrm{N}}(\bar{\psi}\psi\bar{\psi}\psi)] \tag{35}$$

where D_a is the covariant derivative with respect to the local spin connection, G is the gravitational coupling constant, N is the number of fermion species, and the third term is a four-fermion interaction, which at the Fermi surface describes the relevant graviton interaction between pairs of fermions on the Fermi surface.

It is well-known that the physics described by (35) is equivalent to the following Lagrangian

$$\mathcal{L} = \sqrt{\mathrm{g}}[\mathrm{R} + \bar{\psi}\mathrm{D_a}\gamma^{\mathrm{a}}\psi \quad + \bar{\psi}\phi\psi - \frac{\mathrm{N}}{2\mathrm{G}}\phi^2] \tag{36}$$

where $\phi =< \bar{\psi}\psi >$ is the condensate which forms from fermion-graviton interactions.

†The vacuum energy can transmute into massive degrees of freedom, this possibility is currently under investigation by the authors

We will consider graviton exchange between pairs of fermions by expanding about Minkowski spacetime

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} + O(h^2) \tag{37}$$

where $\eta_{\mu\nu}$ is the background Minkowski space metric. For a Dirac fermion, we obtain for a momentum cutoff K_c in the weak curvature limit:

$$i\gamma^\mu p_\mu + \sqrt{\Lambda_0} + \Sigma(p, \sqrt{\Lambda}, G, K_c) = 0 \tag{38}$$

for $i\gamma^\mu p_\mu + \sqrt{\Lambda} = 0$[‡].

We have for a zero bare cosmological constant, $\Lambda_0 = 0$:

$$\sqrt{\Lambda} = \Sigma \tag{39}$$

For gravity for the lowest-order loop we obtain

$$\sqrt{\Lambda} = G_0 \sqrt{\Lambda} F(\sqrt{\Lambda}, K_c) \tag{40}$$

where $F(\sqrt{\Lambda}, K_c)$ is the result of the momentum integration of the Feynman fermion propagators and the cutoff is $K_c = M_{\rm PL}$, where $M_{\rm PL}$ is the Planck mass. This has two solutions: either $\sqrt{\Lambda}$ is zero or

$$\frac{1}{G_0} = F(\sqrt{\Lambda}, K_c) \tag{41}$$

The first solution is the trivial perturbative solution, while the second, nontrivial non-perturbative solution determines $\sqrt{\Lambda}$ in terms of the bare gravitational coupling constant G_0 and the cutoff K_c. The nontrivial solution corresponds to the superfluid condensate state which is the true vacuum state of the system, while the trivial solution corresponds to the normal (false) vacuum state i.e. not the true vacuum state.

The gap equation (41) asymptotically has an exponential dependence on the physical density of states. We have that in an FRW de Sitter background filled with fermions the energy gap will take on the following form, $\Delta = \sqrt{\Lambda}$, separating the two phases. Physically this means that the gap corresponds to the binding energy necessary to form the condensate. The difference between the original vacuum energy and the final vacuum energy is the rest mass of the condensate.

5 Conclusions

We have shown that the vacuum energy in the early universe can become unstable as the attractive fermion-gravitational force and a screened attractive force between positively and negatively charged fermions produces condensates through a phase transition at a critical temperature $T < T_c$. An initial phase of Minkowski flat spacetime with a zero cosmological

[‡] A de Sitter space solution to the non-perturbative gap equation has been obtained for a large N expansion by Inagaki et al. [18].

constant is unstable through a transition to a de Sitter vacuum with an onset of inflation, caused by a non-perturbative vacuum with a large vacuum energy gap $\Delta = \sqrt{\Lambda}$. When the universe ceases to inflate an exponential suppression relaxes the cosmological constant to a small or zero value in the present universe. We described this scenario by analogy with the BCS mechanism associated with the formation of Cooper pairs of fermions by means of the exchange of gravitons.

The non-perturbative mechanism can explain how an initially large vacuum energy (cosmological constant) can be suppressed by a phase transition to a superfluid state of the early universe as the universe expands, leading to a “graceful” exit for inflation.

The non-relativistic toy model we have used to describe the scenario can be extended to a relativistic QFT model of the formation of a vacuum energy gap for fermion condensates in a de Sitter spacetime background.

This scenario explains why a naive *perturbative* calculation of the vacuum energy leads to a nonsensical answer. In contrast to the ground state or vacuum of QED or the standard model, the vacuum associated with gravity is unstable and the instability can only be described by non-perturbative physics.

We note that our model differs from a fundamental scalar field with a tuned potential because the gravitational vacuum ‘knows’ about the composite nature of the condensate unlike for fundamental scalar fields. Our scenario precludes the existence of elementary scalar field particles such as the standard Higgs particle. The Higgs particle is pictured as a composite of fermion-antifermion pairs. The same holds true for the graviton which is described in our picture as a composite condensate of four fermions forming a spin-2 graviton [21].

In future work a more detailed investigation will be carried out of the properties of the gravitational self-energy of the fermions and the role played by the energy gap in early universe cosmology. Since the energy scales during inflation is in the regime of the deconfining phase of QCD it is of interest to see how the gap equation is modified in the presence of free quarks at finite density [19]. It is also important to understand how this mechanism fares with other contributions to the vacuum energy such as composite bosonic degrees of freedom.

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